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BOOK REVIEW

The Classical Stefan Problem: Basic Concepts, Modelling and Analysis. By S. C. Gupta. Elsevier, 2003. 385 pp. ISBN 0444 510869. £66.50.

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The Stefan problem in its simplest form is a macroscopic model for phase changes in a pure material that are brought about purely by heat conduction. Mathematically, the problem lay more or less dormant for more than a century after a simple version of it was first written down by Lamé and Clapeyron in 1831. However, modern technology has stimulated mathematicians of recent decades to increase enormously our understanding of the theoretical intricacies of the solution of both the Stefan problem and many of its generalizations. This understanding is now used routinely in applications ranging from oil extraction to the pricing of American options.

This text follows in the footsteps of the classic volume by Rubinstein (1971), which describes the applicability of the Stefan model and the basic theory up to 1956, and the more theoretical exposition by Meirmanov (1992), whose high point is the existence theory for classical solutions in more than one space dimension. Professor Gupta's book complements Rubinstein (1971) by attempting a wide review of the principal mathematical developments since 1950; readers must not be misled by the title into thinking that only classical solutions are considered as at least half the book concerns weak or generalized solutions. Computational aspects are only mentioned rarely.

A glance at the monumental bibliography by Tarzia (1988) shows that such a review would have been a nearly superhuman task even fifteen years ago. Professor Gupta has tackled it by piecing together around one hundred different mathematical methodologies that have been brought to bear on the Stefan problem yet are widely scattered throughout the literature. Each methodology is illustrated with a piece of analysis, which is often based closely on the original source, together with a verbal description of the relevant modelling. The interested reader is thus well placed to chase up the inevitable theoretical lacunae from the excellent 400-entry bibliography included in this book, many of the references being not at all well-known to the free-boundary community.

The author's approach works quite well in making the book comprehensive, but it is less successful in giving the reader a balanced overview of the many mileposts in the theoretical development. This partly results from the fact that the book does not flow because of numerous misorderings, both of jargon introduced before the definitions thereof, and of complicated analyses that precede more elementary ones by which they could have been illuminated. Moreover the author's bland and uncritical presentation of each of the different methodologies means that many highly specialized techniques and many models of limited practical relevance receive more attention than, for example, the basic theoretical role of similarity solutions or the vastly important subject of alloy solidification. Indeed, although much play is made of the mathematics of mushy regions, all the examples quoted refer to the rarely observed mushes that can form as a result of volumetric heating of pure solids and are manifest as Tyndall stars. The blandness of the text is even more unfortunate in the absence of any reference to option pricing, which would have allowed exciting contacts to be made with the modern theory of stochastic partial differential equations. Short shrift is given to asymptotic methods, whose inclusion would also have enlivened the brief discussion of morphological instability and cusp formation. On the other hand the lengthy and novel chapter on inverse Stefan problems will be very helpful to researchers in many fields.

The publishers have done little justice to the author's achievements in bringing together this large body of knowledge: the English has not been corrected, fonts have not been checked and there are avoidable historical inaccuracies and misspellings of names. It is hard to believe that a text containing a statement (in 8.1) to the effect that Carslaw & Jaeger (1959) considered "the problem of one-dimensional heat conduction in an unbounded medium with constant initial temperature" was ever subject to any vetting.

In summary the book is a valuable compendium of methodologies for the Stefan problem, but it will not help readers wanting to know, say, how Schatz and Biaocchi transformations are related to each other, or what is the real difference between a variational inequality and a quasi-variational inequality. An opportunity has been lost to present the beautiful modern theory of the Stefan problem in a way that emphasizes its undoubted utility to practitioners of many branches of applied science, and to many *JFM* readers.

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